Valued neighborhood search for determination of Pareto-optimal solutions

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***Abstract*—Classical version of the neighborhood search based on exchange operation tests objective function of solution obtained by the exchange operation and depending on the objective function value decides on further progress. When a series of Pareto-optimal solutions of the *p*-location problem is determined, huge number of such operations must be tested and evaluated. As many of exchanges are repeated with the same consequence for the further progress, it is questionable whether the previously obtained information about their success can be taken into account in the decision process. In this contribution, we focus on answering this question. We perform the associated research of influence of information about previous success or failure on effectiveness of the neighborhood search application to determination of a set of Pareto-optimal solutions.**

Keywords—Location problems, conflicting criteria, Pareto front approximation, valued neighborhood

I. INTRODUCTION

Data analysis has become an integral part of many information systems and other software products because it provides very valuable information resulting from real operational data. In the case of the design and optimization of public service systems, which is also the subject of this contribution, it is mainly about the detailed analysis of client requirements, the search for the most important characteristics of the system and the related formulation of the associated objective functions. The main goal of optimization is to ensure better availability and better quality services for system users.

When two or more objectives need to be optimized, the Pareto front, which is an individual collection of non-dominated solutions, must be sought. When evaluating solutions that represent designs for public service systems from the perspective of two contradictory objectives, the Pareto front is a crucial instrument for a system inventor to compromise those objectives. The aforementioned issues are members of the discrete location problem family, which has garnered considerable interest from researchers [1-8, 10, 19-25].

The Pareto front can be determined numerically through the bisection process, in which two integer linear programming runs comprise one phase of the procedure. The bisection procedure is notably time-consuming as a result of the unpredictability of the precise optimization procedure's computational time [9, 11, 12]. Consequently, a sequence of endeavors ensued to utilize heuristic techniques in order to generate a set *NDSS* of solutions that are non-dominated and can be used to approximate the original Pareto front.

The objective function of the solution acquired through the exchange operation is evaluated in the classical version of neighborhood search based on exchange operation. The value of the objective function determines whether further progress is made. After identifying a sequence of Pareto-optimal solutions for the *p*-location problem, an enormous number of these operations must be evaluated and tested. Given the repetitive nature of numerous exchanges that yield identical results regarding subsequent developments, the relevance of previously acquired information regarding their successes in informing the decision process raises doubts. Our contribution is centered around providing an answer to this inquiry.

II. MULTI-CRITERIAL PUBLIC SERVICE SYSTEM DESIGN

A discrete location problem involves the selection of p sites from a set of m candidate locations to minimize a particular criterion value. Thus the set of all feasible problem solutions $\boldsymbol{Y}$ can be defined as $\boldsymbol{Y}=\{\boldsymbol{y}: \boldsymbol{y}\subset\{1, \ldots, m\}, |\boldsymbol{y}|=p\}$.

The quantified criteria of the distinct elements of $\boldsymbol{Y}$ rely on the specific formulation of the locating problem. When it comes to designing a private service system, the main goal is typically to minimize the overall cost of distributing services from service centers to users. The overall expense typically increases in direct proportion to the combined weighted distances between consumers and their nearest service location. When designing a public service system, the situation becomes more intricate because there are multiple perspectives to consider on the system's usefulness. The applicable criteria can be categorized into two kinds known as system criteria and fairness criteria. The system criterion aims to minimize the level of dissatisfaction experienced by the average user of the system, whereas the fairness criterion aims to minimize the level of dissatisfaction experienced by the most disadvantaged minority of system users. The system criterion can be quantified as the mean response time of the system, assuming that a user's request is fulfilled by the closest service center. The fairness requirement can be quantified by the sum of users' requests that fall outside a certain radius R from the closest service center.

Considering random occurrence of the users' demands and limited capacity of the service centers, the nearest available center need not mean the nearest center due to possible occupancy of the nearest center. This situation can be modelled by series $q_1, q_2, \ldots, q_r$ of probability values, where q_k expresses the probability that the k-th nearest service center is the nearest available one [18]. If t_{ij} denotes time necessary for transport of service from a possible service center location

i to a user located at location $j \in \{1, \ldots, n\}$ and if b_j denotes frequency of the demand occurrence at a user's location j, then the system objective function $f_1(\boldsymbol{y})$ can be defined by (1).

$$f_1(\boldsymbol{y}) = \sum_{j=1}^{n}\sum_{k=1}^{r} q_k b_j \min_k \{t_{ij} : i \in \boldsymbol{y}\} \quad (1)$$

In formula (1), the $\min_k$ operation performed on a set of values returns the k-minimum value from the set.

The fairness criterion can be expressed by (2), see [12].

$$f_2(\boldsymbol{y}) = \sum_{j=1}^{n} b_j sign\left(\max\left\{0, \min\left\{t_{ij} - R : i \in \boldsymbol{y}\right\}\right\}\right) \quad (2)$$

The criteria f_1 and f_2 are in conflict, which means that a decrease in one of them is paid for by an increase in the other. It follows that there is no optimal solution, but a usable result of the two-criterion problem can be seen in determining such a set PF of solutions that satisfy clauses (3) and (4).

$$\begin{aligned}&\text{For each } \boldsymbol{x} \in \boldsymbol{Y}, \text{ there exists } \boldsymbol{y} \in PF: \\ &f_1(\boldsymbol{y}) \le f_1(\boldsymbol{x}) \text{ and } f_2(\boldsymbol{y}) \le f_2(\boldsymbol{x})\end{aligned} \quad (3)$$

$$\begin{aligned}&\text{For each pair } \boldsymbol{y}, \boldsymbol{z} \in PF, \text{ it holds that} \\ &\text{either } f_1(\boldsymbol{y}) < f_1(\boldsymbol{z}) \text{ and } f_2(\boldsymbol{y}) \ge f_2(\boldsymbol{z}) \\ &\text{or } f_1(\boldsymbol{y}) \ge f_1(\boldsymbol{z}) \text{ and } f_2(\boldsymbol{y}) < f_2(\boldsymbol{z})\end{aligned} \quad (4)$$

Such a set PF is called a Pareto front. If two solutions $\boldsymbol{x}$ and $\boldsymbol{y}$ satisfy (3), it is said that solution $\boldsymbol{y}$ dominates solution $\boldsymbol{x}$.

III. Approximation of Pareto Front of Public Service System Designs

The frame of the further considered approach is represented by the gradual refinement scheme, which constructs the Pareto front approximation starting with an initial set of two elements and continuing with gradual inserting of the newly determined non-dominated solutions [13, 14, 15, 16].

The initial pair of bordering solutions can be determined by a twin of exact optimization processes so that they minimize one of the applied objectives. This initial part for Pareto front approximation is very simple and does not require enormous computational resources.

The current approximation of the Pareto front (*NDSS*) is kept in the form of ordered set of non-dominated solutions, which is easy to update whenever a candidate for insertion emerges. Complexity of the associated procedure is proportional to the number of *NDSS* members [13, 14, 15, 16].

The gradual refinement scheme subsequently chooses members of the current *NDSS* and submits them to a searching process to achieve an improvement of the Pareto front approximation.

As the Pareto front completion is a hard computational problem, we concentrate our effort on establishing a good approximation of it. The approximating collection of non-dominated solutions (*NDSS*) will be represented by a sequence of *noNDSS* solutions $\boldsymbol{y}^1, \ldots, \boldsymbol{y}^{noNDSS}$ ordered according to increasing values of f_2. Here, the symbol *noNDSS* represents the cardinality of the *NDSS* set and it is assumed to be non-negative integer. To measure the quality of the *NDSS* and the proximity of *NDSS* to *PF*, one may use so-called Area $A(NDSS)$, which is computed according to (5) for any set of solution either *NDSS* or the original *PF*.

$$A(NDSS) = \sum_{k=1}^{noNDSS-1} \left(f_1(\boldsymbol{y}^k) - f_1(\boldsymbol{y}^{noNDSS})\right)\left(f_2(\boldsymbol{y}^{k+1}) - f_2(\boldsymbol{y}^k)\right) \quad (5)$$

Each update of the *NDSS* by a new solution y is followed by a reduction of the *NDSS-Area* and the associated value is bounded from below the *PF-Area*.

As mentioned in previous sections, a good metric for comparing to sets of solutoons is the area formed by the members of *PF* and *NDSS* respectively. The area can be computed easily by the expression (5). To avoid reporting and comparing high values of areas, a simpler coefficient called *gap* can be used. Generally, *gap* can be understood as a relative difference between two values. In our case, it can be expressed by (6).

$$gap = 100 * \frac{A(NDSS) - A(PF)}{A(PF)} \quad (6)$$

IV. Neighborhood Search with Location Rewarding

A The neighborhood search heuristics has proved to be an efficient approach to p-location problem solving, especially when the neighborhood of a current solution is determined by all results of 1-1 exchange operations. The operation, which exchanges one of the chosen locations for one of the non-included, preserves the number p of located service centers. There are several strategies, which can be applied to determination of new current solution. The strategies can be generalized by redefinition of the notion "admissibility" of the candidate for the new current solution and by "range" used for final choice of the new current solution. The process of the neighborhood heuristic can be also limited by time T of its run. The candidate for the new solution is considered to be admissible, if its objective function value is better than the objective function of the current solution at least by the amount A. The range of the new current solution selection is determined by the number N of revealed admissible solutions, from which the best solution is chosen and declared to be the new current solution. This algorithm was applied in the procedure constructing a good approximation of the Pareto front of bi-criterial p-location problem solutions. The presented research is focused on answering the question of how can rewarding of individual locations affect efficiency of the searching process. The generalized neighborhood search heuristics is equipped with a list I of newly established current solutions during one run of the algorithm.

In the below description of the generalized heuristic algorithm, the following denotations are used.

P stands for the set of p selected locations, which determine the current solution.

M stands for the set of all possible service center locations.

$Ex(P, i, j)$ stands for the neighboring solution obtained from P by exclusion of the location $i \in P$ and insertion of the location $j \in M\text{-}P$.

F(*j*) – stands for the reword of the location *j*.

GeneralizedNeighborhoodSearch(*P*, *T*, *N*, *A*, *I*, *F*)

0. Set *OK* = *true*.
1. If $CPU \le T$ and *OK*, then set $n = 0$, $f^* = f(P)$, $i^* = 0$, $Q=M\text{-}P$, $maxR=0$ and continue with step 2. Otherwise return *P* and terminate.
2. While $n<N$ and *OK* perform the following commands for each pair (i,j), $i\in P$, $j \in Q$, and after the cycle has been terminated, go to step 3. If $f(P) - f(Ex(P, i, j)) \ge A$, then set $n = n+1$ and compute $currR=(f(P) - f(Ex(P, i, j)))*F(j)$ if $maxR < currR$, then set $maxR = currR$, $i^* = i$, $j^* = j$.
3. If $i^* > 0$, then define the new current solution $P = Ex(P, i^*, j^*)$, and insert j^* into *I*, else set *OK* = *false*. Continue with step 1.

The above algorithm is subsequently applied to processing of each non-dominated solution of a current approximation of the Pareto front. The processed solutions solves the same bi-criterial *p* – location problem, but used objective function combining the both criteria slightly differs depending on the input solution. This fact evoked an idea that once successful inserted location may be advantages for insertion in other runs of the algorithm. The starting reward of every location is set at the value of 1 at the beginning of whole approximating process and after each run of the *GeneralizedNeighborhoodSearch* the elements of the list *I* are processed and their rewards are updated based on difference *D* of the objective function values of input and output solutions. This updating is performed according to the rule $F(j) = F(j) + D/D^{max}$ for $j \in I$, where D^{max} is theoretically maximal improvement. After these adjustments, forgetting rule is applied to each location $j \in M$. The rule is described by $F(j) = F(j)*(1\text{-}Ro)$, where *Ro* is parameter of the rule.

V. Numerical experiments

The goal of the computational study is to investigate of how can rewarding of individual locations affect efficiency of the searching process in the Pareto front approximation. Quality of Pareto front approximation is evaluated by the resulting gap between the approximation and exact Pareto front areas. The study is carried out on eight benchmarks derived from existing public service systems ensuring emergency medical aid in the higher territorial units - HTU of the Slovak Republic. The exact Pareto fronts were presented in [9, 11] and the associated characteristic are reported in Table 1, where each studied instance corresponds to one row of the table. The first two columns denoted by *m* and *p* contain the problem size. The *NoS* stands for number of solutions forming the Pareto front (*PF*) and the *A*(*PF*) denotes associated *Area*. The list of problem instances contains the HTU of Bratislava (BA), Banská Bystrica (BB), Košice (KE), Nitra (NR), Prešov (PO), Trenčín (TN), Trnava (TT) and Žilina (ZA). In the used input data, all inhabited network nodes represent the set of possible candidate locations of service centers and the possible demand locations as well.

Table 1: Problem sizes and the exact Pareto fronts characteristics

HTU	*m*	*p*	*NoS*	*A*(*PF*)
BA	87	14	34	569039
BB	515	36	229	1002681
KE	460	32	262	1295594
NR	350	27	106	736846
PO	664	32	271	956103
TN	276	21	98	829155
TT	249	18	64	814351
ZA	315	29	97	407293

In this computational study the parameters of *T*, *N* and *A*, were set at values, which were found in the previous research [17, 24]. The values were *T*=0.91 [s], *N*=19, *A*=0.

The experiments were performed for various values of *Ro*. The reached approximation of the Pareto front is characterized by gap, what is difference between areas of the reached approximation and the exact Pareto front expressed in percentage of the area determined by the exact Pareto front.

Table 2: Results of numerical experiments – table of gaps for different values of *Ro*, for the case *Ro* = 0, no update of *F* was performed

Ro	0	0.01	0.02	0.04	0.08
ZA	1.25	1.79	1.79	1.79	1.79
TT	0	0	0.09	0.09	0
TN	0.73	0.73	0.73	0.73	0.73
PO	0.14	0.13	0.14	0.14	0.13
NR	0.99	0.38	0.25	0.25	0.25
KE	1.39	1.21	0.79	1.6	1.24
BB	0.32	0.96	0.52	0.48	0.83
BA	2.18	1.58	1.58	1.58	1.58
sum	7.02	6.8	5.89	6.66	6.56

It must be noted that the computational time of one gradual refinement applied on one instance was limited by 300 seconds.

The experiments reported in this study were performed on a common PC equipped with the Intel® Core™ i7-3610QM CPU@2.30 GHz processor and 8 GB RAM. The algorithms were implemented in Java programming language in the NetBeans IDE 8.2 software.

VI. Conclusions

Discrete location problems can be used in a large variety of important tasks, the goal of which is to minimize either transportation or social costs of service systems providing broad set of users with service from several service centers. Quality of the service system designs may be evaluated by several contradicting criteria.

This scientific paper was focused on such service system designs problem, in which so-called system and fair criteria need to be taken into account. Since creation of the exact Pareto frontier is a hard challenge that requires a lot of computational resources including high process time, the attention of researchers has been paid to the development of various approximate solving approaches.

To determine a good approximation of the original Pareto front, the neighborhood search strategy can be applied as a base for development of different sophisticated algorithms.

The classical form of the neighborhood search algorithm evaluates the objective function of a solution produced through an exchange operation. Based on the value of the objective function, it determines the next steps to be taken. When identifying a sequence of Pareto-optimal solutions for the *p*-location problem, a substantial number of these operations need to be tested and evaluated. Given the repetitive nature of many exchanges and their consistent outcomes, it is uncertain whether the previously acquired information regarding their performance should be considered in the decision-making process. Our objective in this contribution was to provide an answer to this topic. We conducted study on the impact of information on past success or failure on the efficacy of the neighborhood search application for determining a set of Pareto-optimal solutions.

Based on performed computational experiments, we can conclude that we have achieved a very good results in acceptably short computational time. This observation makes proposed method very useful.

Future development in this area could be focused on research of self-learning processes for dynamic parameter setting or on other effective heuristic methods for Pareto front approximation.

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REFERENCES

[1] Ahmadi-Javid, A., Seyedi, P. et al. (2017). A survey of healthcare facility location, Computers & Operations Research, 79, pp. 223-263.

[2] Arroyo, J. E. C., dos Santos, P. M., Soares, M. S. and Santos, A. G. (2010). A Multi-Objective Genetic Algorithm with Path Relinking for the p-Median Problem. In: Proceedings of the 12th Ibero-American Conference on Advances in Artificial Intelligence, 2010, pp. 70–79.

[3] Avella, P., Sassano, A., Vasil'ev, I. (2007). Computational study of large scale p-median problems. Mathematical Programming 109, pp. 89-114.

[4] Brotcorne, L, Laporte, G, Semet, F. (2003). Ambulance location and relocation models. European Journal of Operational Research, 147, pp. 451-463.

[5] Current, J., Daskin, M. and Schilling, D. (2002). Discrete network location models, Drezner Z. et al. (ed) Facility location: Applications and theory, Springer, pp. 81-118.

[6] Doerner, K. F., Gutjahr, W. J., Hartl, R. F., Karall, M. and Reimann, M. (2005). Heuristic Solution of an Extended Double-Coverage Ambulance Location Problem for Austria. *Central European Journal of Operations Research*, 13(4), pp. 325-340.

[7] Drezner, T., Drezner, Z. (2007). The gravity p-median model. *European Journal of Operational Research* 179, pp. 1239-1251.

[8] Gopal, G. (2013). Hybridization in Genetic Algorithms. *International Journal of Advanced Research in Computer Science and Software Engineering*, vol. 3, pp. 403–409.

[9] Grygar, D., Fabricius, R. (2019). An efficient adjustment of genetic algorithm for Pareto front determination. In: TRANSCOM 2019: conference proceedings, Amsterdam: Elsevier Science, pp. 1335-1342.

[10] Ingolfsson, A., Budge, S., Erkut, E. (2008). Optimal ambulance location with random delays and travel times. Health care management science, 11(3), pp. 262-274.

[11] Janáček, J., Fabricius, R. (2021). Public service system design with conflicting criteria. In: IEEE Access: practical innovations, open solutions, ISSN 2169-3536, Vol. 9, pp. 130665-130679.

[12] Janáček, J., Kvet, M. (2021). Emergency Medical System under Conflicting Criteria. In: SOR 2021 Proceedings, pp. 629-635.

[13] Janáček, J., Kvet, M. (2022). Adaptive swap algorithm for Pareto front approximation. In: ICCC 2022: 23rd International Carpathian Control conference, Sinaia, Romania, Danvers: IEEE, 2022, pp. 261-265.

[14] Janáček, J., Kvet, M.. (2022). Repeated Refinement Approach to Bi-objective p-Location Problems. In: INES 2022: Proccedings of the IEEE 26th International Conference on Intelligent Engineering Systems 2022, pp. 41-45.

[15] Janáček, J., Kvet, M. (2022). Pareto Front Approximation using Restricted Neighborhood Search. In: Proceedings of the 40th International Conference on Mathematical Methods in Economics, 2022, Jihlava, pp. 141-147.

[16] Janáček, J., Kvet, M. (2023). Adaptive parameter setting for public service system design. In: Strategic management and its support by information systems 2023, Ostrava: Vysoká škola báňská – Technická univerzita Ostrava, pp. 161-168.

[17] Janáček, J., Kvet, M. (2024). On-line Learning Process for Setting of Heuristic Parameters. In: Mathematical methods in economics 2024, Ústí nad Labem, Czech Republic, september 2024, in print

[18] Jankovič, P. (2016). Calculating Reduction Coefficients for Optimization of Emergency Service System Using Microscopic Simulation Model. In: *17th International Symposium on Computational Intelligence and Informatics*, pp. 163-167.

[19] Jánošíková, Ľ., Kvet, M., Jankovič, P., Gábrišová, L. (2019). An optimization and simulation approach to emergency stations relocation. In Central European Journal of Operations Research 27(3), pp. 737-758.

[20] Jánošíková, Ľ., Jankovič, P., Kvet, M., Zajacová, F. (2021). Coverage versus response time objectives in ambulance location. In: International Journal of Health Geographics 20, pp. 1-16.

[21] Jánošíková, Ľ. and Žarnay, M. (2014). Location of emergency stations as the capacitated p-median problem. In: Quantitative Methods in Economics (Multiple Criteria Decision Making XVII). pp. 117-123.

[22] Kvet, M. (2014). Computational Study of Radial Approach to Public Service System Design with Generalized Utility. In: *Digital Technologies 2014*, Žilina, Slovakia, pp. 198-208.

[23] Kvet, M., Janáček, J. (2022). Directed Search for Pareto Front Approximation with Path-relinking Method. In: Proceedings of the 40th International Conference on Mathematical Methods in Economics, 2022, Jihlava, pp. 212-217.

[24] Kvet, M., Janáček, J. (2024). Swap Heuristic Parameter Sensitivity. In: Mathematical methods in economics 2024, Ústí nad Labem, Czech Republic, september 2024, in print

[25] Marianov, V. and Serra, D. (2002). Location problems in the public sector, *Facility location - Applications and theory* (Z. Drezner ed.), Berlin, Springer, pp 119-150.